

Sterile neutrino signals from supernovae

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We investigate the effects of a mixing of active and sterile neutrinos on the ratios of supernova electron neutrino flux (F_e) and antineutrino flux ($F_{\bar{e}}$) to the total flux of the other neutrino and antineutrino flavours (F_a). We assume that the heaviest (in the normal hierarchy) Standard Model neutrino ν_3 mixes with a sterile neutrino resulting in a pair of mass eigenstates with a small mass gap. Using the density matrix formalism we solve numerically the evolution of neutrino states in the envelope of a supernova and determine the flux ratios F_e/F_a and $F_{\bar{e}}/F_a$ as a function of the active-sterile mixing angle and for the experimentally allowed range of the standard active-active mixing angle θ_{13} .

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Introduction. The neutrino oscillation phenomena observed so far can be explained in terms of three active neutrinos ν_e, ν_μ and ν_τ (for reviews, see [1], [2]). The only exception to this has been, until recently, the muon and electron neutrino and antineutrino oscillation claim from LSND experiment [3], whose consistency with the other data would have required the existence of a fourth neutrino type, a sterile neutrino. The recent results from MiniBooNE experiment [4] seem to refute this claim, however. Hence, as far as the interpretation of the existing experimental data is concerned there seems to be neither need nor too much space for sterile neutrinos [5]. (Notice, however, Ref. [6]).

Nevertheless, it is quite possible that sterile neutrinos do exist anyway. Many electroweak theories beyond the Standard Model predict the existence of right-handed neutrinos, putting neutrinos in this respect on the same footing as the other basic particles, charged leptons and quarks. If the right-handed neutrinos exist they would be sterile under the $SU(2) \times U(1)$ interactions of the Standard Model (SM). Many Grand Unified Theories, e.g. the $SO(10)$ model, include sterile neutrinos, too.

Obviously, sterile neutrinos would have escaped detection if their mixing with the active neutrinos is very small or non-existing. Another reason for their elusiveness could be that they are closely degenerate in mass with the active neutrinos and therefore kinematically indistinguishable from them. The two close-mass eigenstates resulting from the mixing of an active neutrino and a sterile neutrino could have appeared in experiments performed so far as a single neutrino with the normal SM properties, not as two separate states with interactions deviating from the SM.

We have shown earlier that degenerate pairs of neutrinos resulting in mixing of an active and a sterile neutrino, while not separable in laboratory experiments or in the oscillation phenomena studied so far, might reveal themselves by affecting the flux ratios of the active neutrinos ν_e, ν_μ and ν_τ measured at the Earth and originating

in distant astrophysical objects like active galactic nuclei (AGN) [7] and supernovae [8]. Neutrino fluxes from these sources are sensitive to smaller squared mass differences Δm^2 than those detected previously.

In this paper we will return to the subject and study in more detail the effects of sterile neutrinos on the flux of electron neutrinos (F_e) and electron antineutrinos ($F_{\bar{e}}$) from supernovae. We have developed a numerical code that computes the evolution of neutrino energy states in a medium of varying density and determines the flux ratios F_e/F_a and $F_{\bar{e}}/F_a$, where F_a is the total flux of muon and tau neutrinos and antineutrinos, $F_a = F_\mu + F_{\bar{\mu}} + F_\tau + F_{\bar{\tau}}$. We find out that even quite a small mixing of a sterile and an active neutrino might make the ratios F_e/F_a and in particular $F_{\bar{e}}/F_a$ to deviate substantially from the values they obtain in the case of three active neutrinos.

The scheme. We will work in the following active-sterile mixing scheme [7]. We define

$$\nu_\ell = U_{\ell i} \hat{\nu}_i \quad (i = 1, 2, 3), \quad (1)$$

where U is the standard unitary 3×3 matrix parameterized by three rotation angles θ_{12}, θ_{23} and θ_{13} and ν_ℓ ($\ell = e, \mu, \tau$) are the ordinary active neutrinos. We then assume that each state $\hat{\nu}_i$ is accompanied by a sterile state ν_{si} , and that these two states mix (to be called as an ii' -mixing) according to

$$\begin{aligned} \nu_i &= \cos \phi_i \hat{\nu}_i - \sin \phi_i \nu_{si}, \\ \nu'_i &= \sin \phi_i \hat{\nu}_i + \cos \phi_i \nu_{si}, \end{aligned} \quad (2)$$

and form two mass eigenstates with masses m_i and m'_i , respectively. In the limit where the mixing angles ϕ_i or the squared mass differences $\Delta m_{ii'}^2 = m_i^2 - m'^2_i$, or both, are very small, the rotation angles θ_{ij} approach the rotation angles θ_{12}, θ_{23} and θ_{13} of the standard three active neutrino case and $\hat{\nu}_i$'s the three ordinary mass eigenstates.

In this work we will use the following experimental best fit values for the Standard Model mixing angles θ_{12} and

θ_{23} and the squared mass differences [2]:

$$\begin{aligned}\theta_{12} &= 33.7^\circ, \\ \theta_{23} &= 43.3^\circ, \\ \Delta m_{12}^2 &= 7.9 \cdot 10^{-5} \text{eV}^2, \\ \Delta m_{23}^2 &= 2.6 \cdot 10^{-3} \text{eV}^2.\end{aligned}\quad (3)$$

For the mixing angle θ_{13} we use the value $\theta_{13} = 5.2^\circ$, which is the experimental upper limit at 1σ level (at 3σ level $\theta_{13} < 11.5^\circ$) [2].

Solar constraints. Before moving to supernova physics we have to study the restrictions the solar neutrino data sets on the active-sterile neutrino mixing we are considering. The SNO solar neutrino experiment [9] has established boundaries for the relative neutrino fluxes of the active neutrinos arriving from the Sun [2]. Sterile neutrino mixing would affect these ratios, and by requiring that the effects are within the data uncertainties one can constrain the values of active-sterile mixing parameters ϕ_i and $\Delta m_{ii'}^2$. Our numerical study shows that solar neutrino data restrict the $11'$ - and $22'$ -mixings strongly leaving room only for mixings with very small Δm^2 or very small mixing angle, which is in good accordance with earlier work done by other authors (e.g. [10]). The restrictions arise mainly from the low energy part of the flux, where the vacuum oscillation dominates strongly over the matter effects. It turns out that given these tight constraints, the $11'$ - and $22'$ -mixings would not have noticeable effect on supernova neutrino fluxes. For the $33'$ -mixing, in contrast, there is practically no restrictions from the solar neutrino data. This is due to the almost negligible ν_e proportion in the ν_3 and ν_3' states.

This leaves a possibility for large $\Delta m_{33'}^2$ and ϕ_3 , which allows the active-sterile mixing to have a detectable effect on supernova neutrino fluxes as we will show in the following. From now on we will assume that sterile states ν_{s1} and ν_{s2} effectively decouple and that only the sterile state ν_{s3} affects the behaviour of active neutrinos.

Numerical study. We determine the evolution of the states of neutrinos traversing the supernova envelope by solving numerically the density matrix equation

$$\frac{d\rho}{dt} = i[\rho, H], \quad (4)$$

where the Hamiltonian H is given by

$$H = \frac{1}{2E} U \text{diag}(m_1^2, m_2^2, m_3^2, m_3'^2) U^\dagger + \text{diag}(V, 0, 0, V/2), \quad (5)$$

where $V = \sqrt{2} G_F N_e$, and N_e is the number density of electrons [11], assuming that the neutron density is equal to the electron density. The evolution of antineutrino states is obtained similarly, except that the matter term V in the Hamiltonian has the opposite sign.

The flavor decomposition of the neutrino flux as a function of time is given by the diagonal elements ρ_{ii} of the density matrix once the initial fluxes are given.

We will use the typical density profile of a progenitor star as given in [12]. This yields the potential ($r \simeq t$)

$$V(r) = 1.5 \cdot 10^{12} \text{eV} (\text{m}/r)^3. \quad (6)$$

Following [13] we assume that the progenitor star extends to $R = 2.8 \cdot 10^{10}$ m. That is, we take r to vary from the neutrino sphere at about 10^5 m to R .

In Fig. 1 the energy states ν_i ($i = 1, 2, 3$) and ν_3' are plotted as a function of the electron density inside the star, starting from the surface, where the density of matter and subsequently the potential V become zero. At high density the potential V dominates the Hamiltonian (5) and the energy eigenstates are close to the flavour states ν_ℓ and $\bar{\nu}_\ell$ ($\ell = e, \mu, \tau, s$). At zero density the states equal to the mass eigenstates (2).

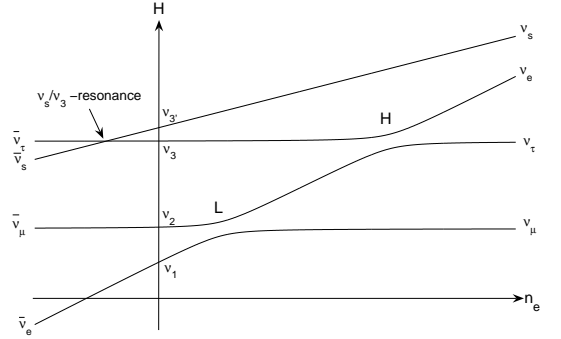


FIG. 1: Neutrino eigenstates in matter in the presence of $33'$ -mixing. The L -resonance is always adiabatic, whereas behavior in the H - and $33'$ -resonances depend on the value of the corresponding mixing angles θ_{13} and φ .

We assume the initial neutrino and antineutrino fluxes to be as given in [14]. The authors of that reference have developed a model for supernova explosion and based on this model they give the fluxes of ν_e , $\bar{\nu}_e$ and $\nu_\mu \simeq \nu_\tau \simeq \bar{\nu}_\mu \simeq \bar{\nu}_\tau \equiv \nu_x$ as a function of neutrino energy. More specifically, we will use in our analysis the ratios $F_e^0 : F_{\bar{e}}^0 : F_x^0 = 4 : 2.3 : 1.4$, which correspond to the fluxes at the energy 10 MeV where the intensity of the neutrino radiation is largest. Of course, the fluxes, F_x^0 in particular, will be hard to measure within a single energy bin [16]. Nevertheless, the flux ratios at the peak energy 10 MeV turn out to be very close to the integrated fluxes, with just slightly overweighting the fraction of electron neutrinos, so that they can be reliably used as input fluxes. Sterile neutrinos are of course not produced in supernovae but they are created, as a result of their assumed mixing with the standard model neutrinos, during the traverse of neutrinos the medium of the star.

Results. Let us first assume that sterile neutrinos are absent and study the system with the three active neutrinos and their antineutrinos only. By using these as

initial conditions we solved numerically the density matrix equation and determined the flux ratios F_e/F_a and $F_{\bar{e}}/F_a$. Allowing the mixing angle θ_{13} to vary in the range $0 \leq \theta_{13} \leq 5.2^\circ$ and fixing the other mixing parameters to their best-fit values (3) the flux ratios in the case of three active neutrinos are found to vary within the ranges

$$\begin{aligned} 0.17 < F_e/F_a < 0.29, \\ 0.24 < F_{\bar{e}}/F_a < 0.26. \end{aligned} \quad (7)$$

The sensitivity of these ratios on the mixing angle θ_{13} is due to the fact that the adiabaticity of the so called H -resonance at the crossing of the energy states ν_2 and ν_3 (see Fig. 1) depends on the value of this angle. When $\theta_{13} = 0$, the behavior of the system is fully non-adiabatic, meaning that the flux of electron neutrinos, which is the largest one initially, ends to the energy state ν_2 , which contains less of ν_a ($= \nu_\mu + \nu_\tau + \bar{\nu}_\mu + \bar{\nu}_\tau$) than the energy state ν_3 . This corresponds to the largest values of the ratios F_e/F_a and $F_{\bar{e}}/F_a$. The smallest values of these ratios are obtained when θ_{13} attains its largest experimentally allowed value. In this case the H -resonance is adiabatic and the initial electron neutrino flux ends into the state ν_3 , which has a very small ν_e component, $|\langle \nu_e | \nu_3 \rangle|^2 = \sin^2 \theta_{13} = 0.008$.

We now add into the scheme a sterile neutrino. We assume that the sterile neutrino mixes with the active neutrinos in such a way that the states ν_1 and ν_2 remain intact, that is, their flavor decomposition is the same as in the Standard Model and consist only of active flavors, while the third state ν_3 becomes a superposition of the active neutrinos and the sterile neutrino according to (2). The state ν_3 is assumed to be close in mass to the new heavier state ν'_3 orthogonal to it. In our analysis we will assume $m_3'^2 - m_3^2 = 10^{-6} \text{ eV}^2$. Increasing $\Delta m_{33'}^2$ from this value would not change our results. For lower values of $\Delta m_{33'}^2$, the effect of active-sterile mixing decreases and will be washed out at $\Delta m_{33'}^2 \approx 10^{-11} \text{ eV}^2$, where our results would coincide with the SM result. As ν'_3 is heavier than ν_3 , the energy levels corresponding to these two states cross in the antineutrino sector, not in the neutrino sector (see Fig. 1).

In Fig. 2 we present the flux ratios F_e/F_a and $F_{\bar{e}}/F_a$ as a function of the active-sterile mixing angle ϕ for the experimentally allowed range of the angle θ_{13} , $0 \leq \theta_{13} \leq 5.2^\circ$. In what follows we will explain the behavior of the ratios qualitatively.

The both ratios are larger for $\theta_{13} = 0^\circ$ than for $\theta_{13} = 5.2^\circ$. This feature is easy to understand in the following way. When $\theta_{13} = 0$, the H -resonance in the neutrino sector is non-adiabatic and none of the electron neutrinos created in the supernova core ends up to the state ν_3 and the state ν_3 will be populated solely by the less intense ν_τ component of the initial flux. The original electron neutrinos end up to the state ν_2 whose ν_e component is much larger than that of ν_3 , where the original electron neutrinos will end to in the case of $\theta_{13} = 5.2$. As to the

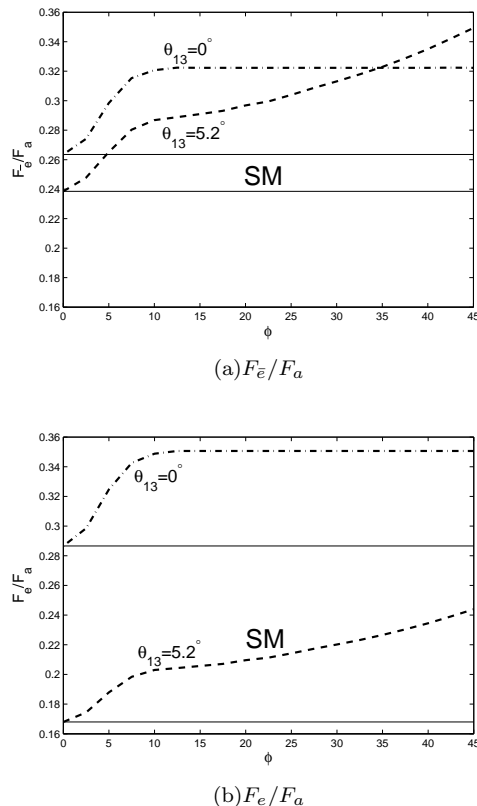


FIG. 2: Ratio of (a) $\bar{\nu}_e$ (b) ν_e and nonelectronic active fluxes as a function of sterile mixing angle ϕ . Results corresponding to the minimum and maximum values of θ_{13} allowed by experiment are shown. The region denoted by SM is the allowed range in the SM case of three active neutrinos. Here the neutrino energy is taken as $E = 10 \text{ MeV}$.

ratio $F_{\bar{e}}/F_a$, the flux $F_{\bar{e}}$ does not depend on the value of θ_{13} , but F_a is larger in the case of $\theta_{13} = 5.2^\circ$ than in the case of $\theta_{13} = 0^\circ$ as then the original electron neutrino flux ends to the state ν_3 , where there are more muon and tau neutrinos than in the state ν_2 .

The $33'$ -mixing affects the ratios F_e/F_a and $F_{\bar{e}}/F_a$ in two ways. Firstly, the mixing makes the flavor decomposition of the energy eigenstate ν_3 to change from its SM form as it replaces the active flavors partly by the sterile flavor. As there is only a tiny electron neutrino (antineutrino) component in ν_3 ($\bar{\nu}_3$), this would affect mainly the flux F_a and makes the value of the ratios F_e/F_a and $F_{\bar{e}}/F_a$ larger than in the Standard Model case. The effect of course increases with increasing active-sterile mixing angle ϕ . This explains the growth of the ratios when ϕ shifts from zero to about 12° .

The second effect of the active-sterile mixings is related to the possible level crossing in the resonance transition region of the $\bar{\nu}_3$ and $\bar{\nu}'_3$ antineutrino states. For a small mixing angle ϕ , the $33'$ -transition is non-adiabatic and the level crossing takes place. That is, all $\bar{\nu}_\tau$'s created in the supernova will end up to the state $\bar{\nu}'_3$, which has only

a tiny sterile content, proportional to $\sin^2 \phi$.

When $\phi > 12^\circ$ the $33'$ -resonance is fully adiabatic. In this case the original ν_τ flux will go into the state ν'_3 , which has a sterile fraction proportional to $\cos^2 \phi$. When ϕ increases, the $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ components of $\bar{\nu}'_3$ will become larger, which will make F_a to *increase*. What will happen to the ratios depends, however, on the adiabaticity of the H-resonance. When $\theta_{13} = 0$ the H-resonance is non-adiabatic, the occupation of the state ν_3 originates from the original ν_τ flux. When the angle ϕ increases, the ν_μ and ν_τ components of ν_3 will become smaller and the sterile part will become larger. This will make F_a to *decrease*. As the original flux of tau neutrinos and antineutrinos are assumed to be equal, the net effect will be that F_a remains practically constant for $\phi > 12^\circ$. As the electron neutrino and antineutrino fluxes are quite insensitive to the active-sterile mixing, this implies that the values of the ratios F_e/F_a and $F_{\bar{e}}/F_a$ saturate to the constant values for $\phi > 12^\circ$ and $\theta_{13} = 0$. According to Fig. 2, the saturation values are $F_e/F_a = 0.35$ and $F_{\bar{e}}/F_a = 0.32$, to be compared with the Standard Model values 0.29 and 0.26, respectively.

When $\theta_{13} = 5.2^\circ$, the H-resonance is adiabatic. Neutrinos created originally as ν_e will in this case end up to the state ν_3 . In contrast with the previous case, now the ratios F_e/F_a and $F_{\bar{e}}/F_a$ will not saturate after the active-sterile mixing has achieved the value $\phi = 12^\circ$ but will keep increasing with ϕ . The difference to the previous case is that now the state ν_3 will be more occupied than the state ν'_3 as the original electron neutrino flux is assumed to be larger than the tau antineutrino flux. For the maximal $33'$ -mixing, i.e. for $\phi = 45^\circ$, one has $F_e/F_a = 0.24$ and $F_{\bar{e}}/F_a = 0.35$, to be compared with the SM values 0.17 and 0.24, respectively. Note that $F_{\bar{e}}/F_a$ becomes insensitive to the angle θ_{13} when the active-sterile mixing angle has the value about 35° .

Discussion and summary. As one can infer from the Fig. 2, the ratio $F_{\bar{e}}/F_a$ would give more information on the active-sterile mixing than F_e/F_a taking the uncertainty of the value of the mixing angle θ_{13} into account. Given the value of $F_{\bar{e}}/F_a$ from measurements, the angle ϕ will be limited in general into quite a narrow range of values. Also, for $\phi \gtrsim 5^\circ$ the $F_{\bar{e}}/F_a$ would deviate from the SM

prediction for all allowed values of θ_{13} , in contrast with F_e/F_a .

As for the uncertainties in the initial neutrino fluxes, the differences between the SM case and sterile case arise only due to the $33'$ -mixing in the antineutrino part. So only the initial flux of $\bar{\nu}_\tau$ will affect the ratios by possibly changing the value of F_a .

It is worthwhile to make a remark concerning the effect of the Earth on the fluxes. As there is only a tiny electron neutrino component in the state ν_3 , the effects of neutrino-matter interactions in the Earth will not differ much from those in the SM case. The active-sterile mixing will only slightly change the fluxes of non-electron flavour neutrinos measured at the opposite side of the Earth. The most visible effect would be a distortion of the ν_l , ($l \neq e, \bar{e}$) energy spectra, which could possibly be measured in future supernova neutrino experiments [15].

In summary, we have considered the situation where neutrinos appear as three pairs of states, where each pair originates in a mixing of an active neutrino and a sterile neutrino and its components are so close in mass that they are not distinguished as different particles in the present experiments. We have shown that this kind of structure might manifest itself through the fractions of electron neutrinos and antineutrinos in the supernova neutrino flux, i.e. through the ratios F_e/F_a and $F_{\bar{e}}/F_a$. Apart from the active-sterile mixing angle ϕ , the deviations of these ratios from their Standard Model values are sensitive also to the mixing angle θ_{13} for which an upper limit only is known at the moment. Only an active-sterile mixing associated with the heaviest mass state ν_3 can have measurable effects, since the solar neutrino data restrict the mixing parameters in the case of the two other states too small to lead any observable changes in the supernovae neutrino fluxes.

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